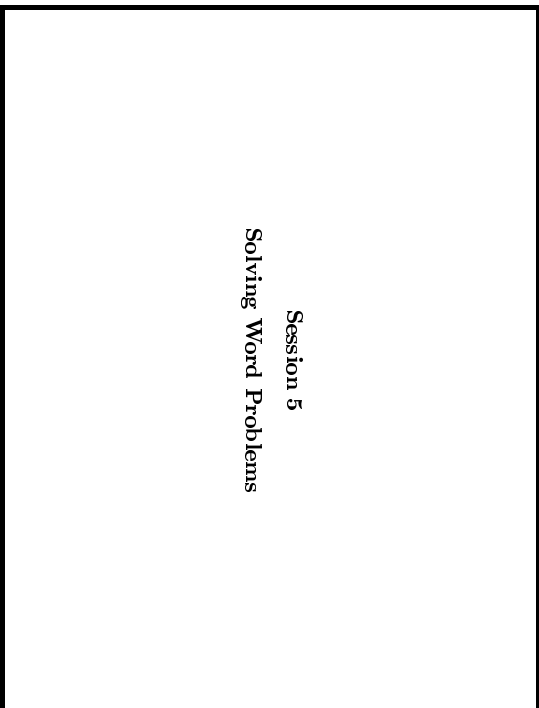


Slide 1

Session 5
Solving Word Problems



Slide 2

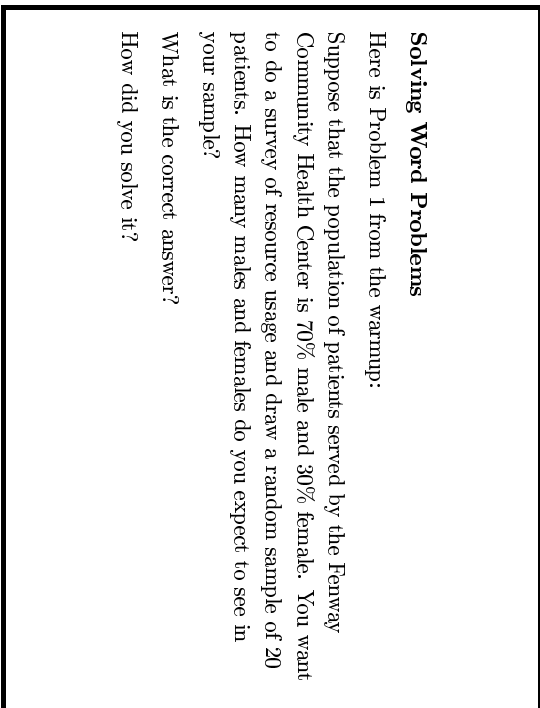
Solving Word Problems

Here is Problem 1 from the warmup:

Suppose that the population of patients served by the Fenway Community Health Center is 70% male and 30% female. You want to do a survey of resource usage and draw a random sample of 20 patients. How many males and females do you expect to see in your sample?

What is the correct answer?

How did you solve it?



Slide 3

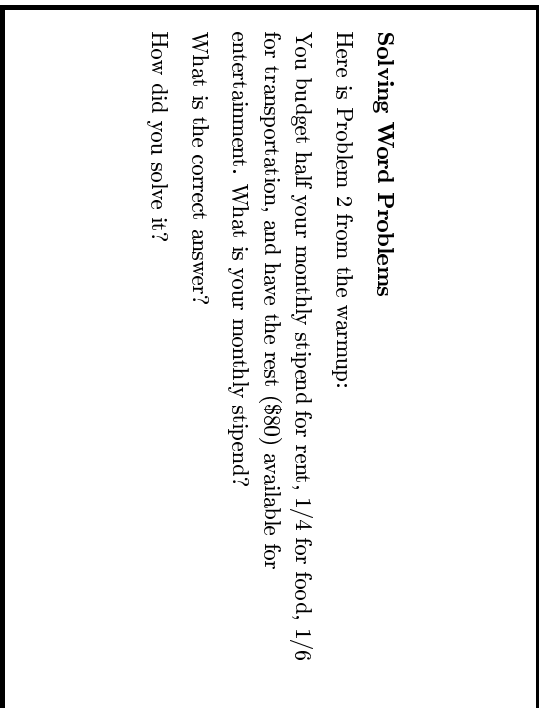
Solving Word Problems

Here is Problem 2 from the warmup:

You budget half your monthly stipend for rent, $\frac{1}{4}$ for food, $\frac{1}{6}$ for transportation, and have the rest (\$80) available for entertainment. What is your monthly stipend?

What is the correct answer?

How did you solve it?

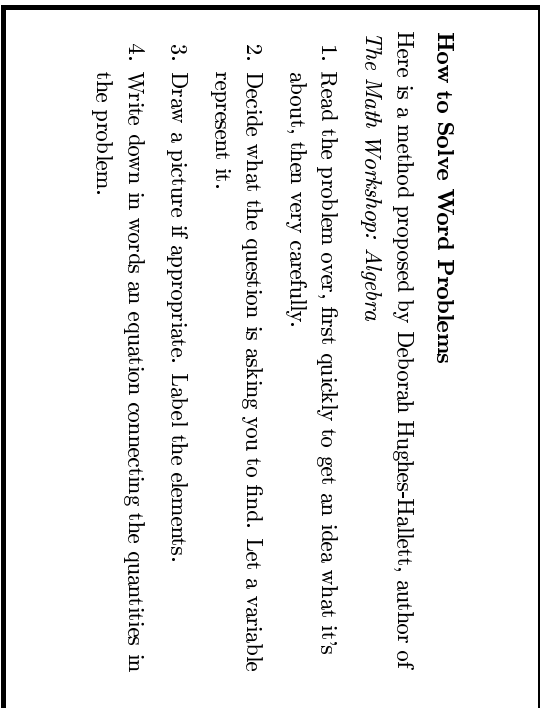


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How to Solve Word Problems

Here is a method proposed by Deborah Hughes-Hallett, author of *The Math Workshop: Algebra*

1. Read the problem over, first quickly to get an idea what it's about, then very carefully.
2. Decide what the question is asking you to find. Let a variable represent it.
3. Draw a picture if appropriate. Label the elements.
4. Write down in words an equation connecting the quantities in the problem.



How to Solve Word Problems

5. List any formulas that might help.
6. Write expressions for unknown quantities in terms of the variable.
7. Write the equation in symbols, by expressing unknown quantities in terms of the variables.
8. Solve the equation.
9. Check that your solution is reasonable, and that it answers the question asked by the problem.

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Solving Word Problems: Example

4. An equation in words:

new solution = salt + old water + added water
added water = new solution - salt - old water

5. Useful formulas:

salt = 8% of old solution and 5% of new solution
new solution = salt / 0.05
old water = 92% of old solution
old solution = 75 ounces

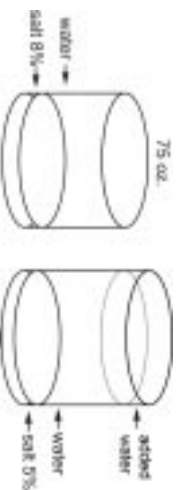
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Solving Word Problems: Example 1

How many ounces of pure water must be added to 75 ounces of an 8% salt solution to make a 5% salt solution?

1. Read the problem carefully.
2. You are asked to find the added water needed to obtain a 5% solution. We will call added water aw .

3. A picture:



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Solving Word Problems: Example 1

6. Express unknown quantities in terms of the variable:

added water = new solution - salt - old water amount
added water = salt / 0.05 - salt - 92% of old solution
added water = salt / 0.05 - .08 (75 ounces) - .92 (75 ounces)
added water = salt / 0.05 - 75 ounces
added water = .08 (75 ounces) / 0.05 - 75 ounces

Solving Word Problems: Example 1

7. Write equation(s) in symbols:

$$aw = ns - s - aw$$

$$aw = s/.05 - s - .92os$$

$$aw = \frac{.08 * os}{.05} - 0.08os - .92os$$

$$aw = \frac{.08 * os}{.05} - os$$

8. Solve the equation:

$$aw = \frac{.08 * 75}{.05} - 75$$

$$aw = 120 - 75$$

$$aw = 45 \text{ ounces of water}$$

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Solving Word Problems: Example

9. **Check:** We originally had 75 ounces of 8% salt solution. This was $(.08)(75) = 6$ ounces of salt and $(.92)(75) = 69$ ounces of water. We added 45 ounces of water to 69 ounces of water, for a total of 114 ounces of water. We still have 6 ounces of salt, so we have 120 ounces of solution. Is 6 ounces 5% of 120 ounces? Yes! Did we answer the question that was asked? Yes, we know how many ounces of water to add to the original solution.

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Solving Word Problems: Example 2

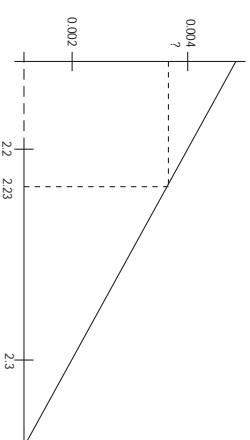
You have done a statistical test and have a value for your test statistic of 2.23. You are lying on the beach with a statistical table, but no computer to find the exact p-value for your test statistic. From the table, you do know the p-values for these test statistics:

Test Statistic	p-value
2.2	0.004
2.3	0.002

Assuming a linear relationship, what is the p-value for your test statistic?

Solving Word Problems: Example 2

1. Read.
2. You are asked to find the p-value (call it y).
3. A picture:



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Slide 13**Solving Word Problems: Example 2**

4. We want to find a point lying on a straight line through (2,2, 0.004) and (2,3, 0.002).
5. $m = \frac{y_2 - y_1}{x_2 - x_1}$, $y_1 = mx_1 + b$
6. p-value = slope(test statistic) + intercept
7. $m = \frac{0.002 - 0.004}{2.3 - 2.2} = \frac{-0.002}{0.1} = -0.02$

Solving Word Problems: Example 2

8.

$$0.004 = -0.02(2.2) + b$$

$$0.004 = -0.044 + b$$

$$0.048 = b$$

$$y = -0.02x + 0.048$$

$$y = -0.02(2.23) + 0.048$$

$$y = 0.0034$$

9. The p-value for your test statistic is 0.0034. This is reasonable because it falls between 0.002 and 0.004. (Check the picture.) We can also check the formula for the line by plugging in the values for x_2 and y_2 .

Slide 15**Solving Word Problems: Example 3**

You are doing survey research about health practices. You send a survey to everyone on the HMO membership list. 70% of those who receive the survey throw it out immediately. Of the rest, only 10% return the survey. You receive 1500 surveys back. How many people are on the membership list?

Slide 16**Solving Word Problems: Practice**

Let x = the number of people on the membership list

Number returning survey = 1500

Number returning survey = 10% of those who didn't throw surveys away

Number who threw surveys away = 70% of recipients

Number who didn't throw surveys away = 30% of recipients

$$= \frac{30}{100}x = 0.3x$$

Solving Word Problems: Example 3

$$\begin{aligned} \text{Number returning survey} &= 10\%(0.3x) \\ &= \frac{10}{100}(0.3x) = 0.1(0.3x) \\ &= 0.03x \\ 1500 &= 0.03x \\ \frac{1500}{0.03} &= x \\ x &= 50,000 \text{ members} \end{aligned}$$

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Solving Word Problems: Signals

Is is a signal for equals

Here are some words that signal **Addition**

- Added to
- More than
- The sum of
- Increased by
- The total of

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Solving Word Problems: Example 3

Let's check:

$x = 50,000$ members

70%, or $(.70)(50,000) = 35,000$ throw it out immediately.

Of the remaining 15,000, if 10% return the survey, we should have $(.10)(15,000) = 1,500$ surveys back. Correct!

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Solving Word Problems: Signals

Here are some words that signal **Subtraction**

- Minus
- Less than
- Decreased by
- The difference between

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Solving Word Problems: Signals

Here are some words that signal **Multiplication**

- Times
- Of
- The product of
- Multiplied by
- Twice

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Solving Word Problems: Practice with Signals

1. Find a number whose double is eight more than the result of subtracting the original number from 25.
2. Find two numbers whose difference is eight, such that the larger number is sixteen less than three times the smaller number.

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Solving Word Problems: Signals

Here are some words that signal **Division**

- Divided by
- The quotient of
- The ratio of

Here are some words that signal **Powers**

- The square of
- The cube of