# Exercise 6 Solutions

## Solve

1. $8! - 4! = 40320 - 24 = 40,296$

2. $\frac{8!}{4!} = \frac{40320}{24} = 8 \cdot 7 \cdot 6 \cdot 5$

3. $\binom{8}{4} = \frac{8!}{4!4!} = \frac{1680}{4 \cdot 3 \cdot 2} = 70$

4. How many ways can you choose 6 objects from among 10?

   $\binom{10}{6} = \frac{10!}{6!4!} = 210$

5. How many ways can you choose 3 objects from among 3?

   $\binom{3}{3} = 1$

6. How many ways can you choose 1 object from among 5?

   $\binom{5}{1} = 5$

7. How many ways can you choose $n - 1$ objects from among $n$?

   $\binom{n}{n-1} = \frac{n!}{(n-1)!} = \frac{n(n-1)!}{(n-1)!} = n$

8. $\sum_{i=1}^{5} \frac{X_i}{n}$, where $X_i$ are the first 10 positive integers (be careful!)

   $\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \frac{4}{n} + \frac{5}{n} = \frac{15}{n}$

9. Calculate $\sum_{j=1}^{6} 2j \cdot \ln j$

   $2(1)\ln(1) + 2(2)\ln(2) + 2(3)\ln(3) + 2(4)\ln(4) + 2(5)\ln(5) + 2(6)\ln(6)$
   $= 0 + 2.7726 + 6.5917 + 11.0904 + 16.0944 + 21.5011 = 58.05$

10. Solve for $x$ and check your solution:

$$\sum_{i=0}^{2} ix^i = 5$$

   $0x^0 + 1x^1 + 2x^2 = 5$
   $2x^2 + x - 5 = 0$
   $x = \frac{-1 \pm \sqrt{1 - 4(2)(-5)}}{4}$
   $x = 1.351 \text{ or } -1.851$
   Check: $2(1.351)^2 + 1.351 - 5 = 0$? Yes! $2(-1.851)^2 - 1.851 - 5 = 0$? Yes!

11. Calculate $\displaystyle\sum_{k=0}^{3} \binom{3}{k} (0.4)^k (0.6)^{3-k}$

$\binom{3}{0}(.4)^0(.6)^{3-0} + \binom{3}{1}(.4)^1(.6)^{3-1} + \binom{3}{2}(.4)^2(.6)^{3-2} + \binom{3}{3}(.4)^3(.6)^0$

$= .6^3 + 3(.4)(.6^3) + 3(.4^2)(.6) + .4^3$

$= 0.216 + 0.432 + 0.288 + 0.064 = 1$

12. Simplify the following expression:

$$\sum_{i=0}^{2}(x+iy)^i$$

$(x+0y)^0 + (x+y)^1 + (x+2y)^2$

$= 1 + x + y + x^2 + 4y^2 + 4xy$

$= x^2 + 4y^2 + 4xy + x + y + 1$