Exercise 5 Solutions

Solve these problems:

1.(a) You are conducting a study and participants must pass 2 screenings, first by a medical doctor and then by a psychiatrist, in order to be eligible. If the MD's reject one potential subject out of 12 and the psychiatrists reject 40% of remaining potential subjects, what percent of potential subjects make it past both screenings?
(b) If it costs \$300 each time an MD examines a potential subject and \$1200 each time a psychiatrist examines one, would it be cheaper to run the screenings the other way around?

(a) If the MD rejects 1/12, then 11/12 pass the MD screening. The psychiatrist rejects 40%, so 60% of the remaining 11/12 pass both screenings. $\frac{11}{12} \times 0.6 = 0.55$, or 55%.
(b) Assume we screen 100 patients. Using the current system, the cost would be

$$(100 \times \$300) + (\frac{11}{12} \times 100 \times \$1200) = \$30,000 + \$110,004, \text{ or } \$140,004$$

If the psychiatrist went first and we assumed she rejected patients at the same rate as before, the cost would be

$$(100 \times \$1200) + (60 \times \$300) = \$120,000 + \$18,000, \text{ or } \$138,000$$

It would be cheaper to run the screenings the other way around.

2. "The top runners in the Boston Marathon cover the 26-mile distance in 2 hours and 15 minutes. If they average 12 miles per hour over the level part of the course but only 8 miles per hour in the infamous Heartbreak Hill area, how many of the 26 miles are level?"

Let L = miles run on level ground and H = miles run on hilly ground. There are 26 miles total, so $L + H = 26$, and $H = 26 - L$.

$$\begin{aligned}
\frac{L}{12} + \frac{H}{8} &= 2.25 \\
\frac{L}{12} + \frac{26 - L}{8} &= 2.25 \\
\frac{2L}{24} + \frac{3(26 - L)}{24} &= 2.25 \\
\frac{2L + 3(26) - 3L}{24} &= 2.25 \\
78 - L &= 24(2.25) \\
-L &= 54 - 78 \\
L &= 24
\end{aligned}$$

24 of the miles are hilly.

Here's another solution. We cover the 26 miles in 2.25 hours, so the average speed is 26/2.25, or 11.556 miles per hour. Let Π = the proportion of time running level.

$$11.556 = \Pi(12) + (1 - \Pi)(8)$$

$$\begin{aligned} 11.556 &= 12\Pi + 8 - 8\Pi \\ 3.556 &= 4\Pi \\ \Pi &= 0.889 \end{aligned}$$

So 89% of the time is spent running level. 89% of 2.25 hours is 2 hours, and 2 hours at 12 miles per hour is 24 miles running level.

3. Thermal pollution is a serious problem in American rivers. Water has a "heat content" equal to the product of its temperature and volume, and the temperature of a mixture can be found by dividing the sum of its heat contents by its total volume. A river has a flow of 100 million gallons per day and a normal temperature of 70 degrees F. What is the maximum volume of water at 130 degrees F that a nuclear power plant can be allowed to discharge into the river per day if a change in the river temperature of more than 8 degrees F will wreck the balance of the ecosystem?

We know that the total temperature will be a mixture of 2 parts, and that each part has a heat content equal to its temperature times its volume: $H = TV$, and $T = \frac{H}{V}$. So the temperature of the total can be written as follows, where T is the desired final temperature of 78 degrees:

$$\begin{aligned} T &= \frac{H_1 + H_2}{V_1 + V_2} \\ H_1 &= T_1V_1 = (70 \text{ degrees})(100 \text{ million gallons}) \\ H_2 &= T_2V_2 = (130 \text{ degrees})(V_2) \\ 78 &= \frac{(70)(100) + (130)V_2}{100 + V_2} \\ 78(100 + V_2) &= 7000 + 130V_2 \\ 7800 + 78V_2 &= 7000 + 130V_2 \\ 800 &= 52V_2 \\ V_2 &= 15.38 \text{ million gallons/day} \end{aligned}$$