Exercise 4 Solutions

Solve:

1. $\ln 3.4 = 1.2238$

2. $e^{4.2} = 66.686$

3. $\ln(e^{10.7}) = 10.7$

4. $e^{(\ln 10.7)} = 10.7$

5. $\ln(4^3) = 3\ln(4) = 4.1589$

6. $\ln(21 \times 4) = \ln(21) + \ln(4) = 3.0445 + 1.3863 = 4.4308$

7. $\ln(\frac{3}{4}) = \ln(3) - \ln(4) = 1.0986 - 1.3863 = -0.2877$

Graph:

8. $y = 2 + 2e^{2x}$ (See last page)

9. $y = 3\ln(x + 5)$ (See last page)

Solve the following equations and check your solutions:

10.
$$
\begin{aligned}
\ln(x+5) &= 2 \\
x+5 &= e^2 \\
x &= e^2 - 5 \\
&= 2.3891
\end{aligned}
$$

11.
$$
\begin{aligned}
e^{2x-3} + 5 &= 10 \\
2x - 3 &= \ln(5) \\
2x &= \ln(5) + 3 \\
x &= \frac{\ln(5) + 3}{2} \\
&= 2.3047
\end{aligned}
$$

12.

$$\begin{aligned}
\ln(1 + e^{-3x}) &= 7 \\
1 + e^{-3x} &= e^7 \\
e^{-3x} &= e^7 - 1 = 1095.63 \\
-3x &= \ln(1095.63) = 6.999 \\
x &= \frac{6.999}{3} = -2.33
\end{aligned}$$

13.

$$\begin{aligned}
5 + exp(-2x) &= \ln(3) \\
e^{-2x} &= \ln(3) - 5 = -3.90 \\
-2x &= \ln(-3.90)
\end{aligned}$$

This cannot be solved, because the natural log of a negative number is undefined.

14. Here is a table relating vaccination rates and disease-free rates for one year:

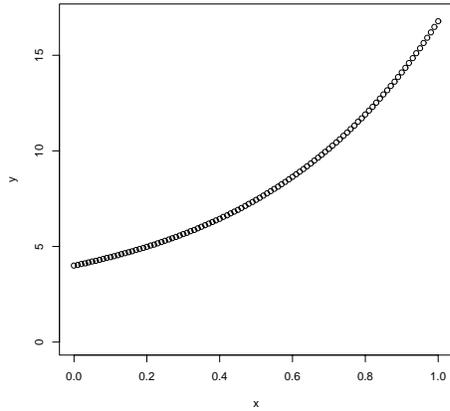|		Vaccinated	
		Yes	No
Disease-	Yes	77	13
Free	No	8	2

Estimate the odds ratio for the association between vaccination and disease-free rates and determine the 95% confidence interval for the odds ratio.

$$\begin{aligned}
\widehat{OR} &= \frac{(77)(2)}{(13)(8)} = \frac{154}{104} = 1.48 \\
\ln(\widehat{OR}) &= 0.393 \\
\widehat{sd}(\ln(\widehat{OR})) &= \sqrt{\frac{1}{77} + \frac{1}{13} + \frac{1}{8} + \frac{1}{2}} = 0.846 \\
95\%\text{CI } \ln(\widehat{OR}) &= 0.393 \pm 1.96(0.846) \\
&= 0.393 \pm 1.657 \\
&= (-1.265, 2.050) \\
95\%\text{CI } \widehat{OR} &= (e^{-1.265}, e^{2.050}) \\
&= (0.282, 7.766)
\end{aligned}$$

Graph:

$y = 2 + 2e^{2x}$

x	y
-1	2.7
-0.5	2.7
0	4
0.5	7.4
1	16.8

$y = 3\ln(x + 5)$

x	y
-4	0
-2	3.3
0	4.8
2	5.8
4	6.6