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Math Workshop

Greatest Hits from On-Line Modules

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Doing Math: Rounding

We can *round* numbers to make them easier to understand and compare. It is important not to imply that numbers have great precision, when we simply have failed to round off. It's best to work with extra decimal places while doing calculations, then round off the answer as the last step in solving a problem.

$$\frac{19}{30} = 0.6333 \qquad \frac{7}{11} = 0.6363$$

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Doing Math: How to Round

1. Look at the digit to the right of the one you want to keep.
2. Is it 5 or greater? Then increase the digit you want to keep by 1.
3. Is it 4 or smaller? Then leave the digit you want to keep alone.
4. If the "keeper" digit is to the right of the decimal, drop all digits farther to the right.
5. If the "keeper" digit is to the left of the decimal, change all digits farther to the right to 0.

Example: Round to the nearest hundredth:

$$0.6363 \rightarrow 0.64 \qquad 0.6333 \rightarrow 0.63$$

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Practice Rounding:

855.23 round to tens

round to tenths

round to ones
(nearest integer)

round to hundreds

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Doing Math: Multiplying and Dividing Negative Numbers

First, do the math as if the signs were positive.

If signs are the same, the answer is positive.

If signs are opposite, the answer is negative.

Examples:

$$(-5) \times (+3) = -15 \rightarrow \textit{signs opposite}$$

$$(-5) \times (-5) = 25 \rightarrow \textit{signs the same}$$

$$(-3) \times 2 \times 2 = -12$$

$$(-4) \times 3 \times (-3) = 36$$

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Practice with Negative Numbers: Hand Calculations

$-4 + 3 - 2 + 5 - 8 =$

$12 - 15 + 18 - 27 + 17 =$

$-12 \times 2 =$

$-11 \cdot -2$ ($\cdot$ means multiply) $=$

$-3 * 4 * -2$ (* means multiply) $=$

$(7)(-7)(2)$ (( ) means multiply) $=$

$(2)(-2)(-3)(3) =$

$(-3)(-2)(-3)(2) =$

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Practice with Negative Numbers: Using a Calculator

$-4 + 3 - 2 + 5 - 8 =$

$12 - 15 + 18 - 27 + 17 =$

$-12 \times 2 =$

$-11 \cdot -2 =$

$-3 * 4 * -2 =$

$(7)(-7)(2) =$

$(2)(-2)(-3)(3) =$

$(-3)(-2)(-3)(2) =$

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Useful Rules - "The Properties of Real Numbers"

$$\left.\begin{aligned} a + b &= b + a \\ 3 + 2 &= 2 + 3 \\ 5 &= 5 \end{aligned}\right\} \text{The commutative property of addition}$$

$$\left.\begin{aligned} a * b &= b * a \\ (3)(-2) &= (-2)(3) \\ -6 &= -6 \end{aligned}\right\} \text{The commutative property of multiplication}$$

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Useful Rules - "The Properties of Real Numbers"

$$\left.\begin{array}{c}(a+b)+c=a+(b+c)\\(3+4)+5=3+(4+5)\\7+5=3+9\\12=12\end{array}\right\}\text{The associative property of addition}$$

$$\left.\begin{array}{c}(a*b)*c=a*(b*c)\\(3*4)*5=3*(4*5)\\12*5=3*20\\60=60\end{array}\right\}\text{The associative property of multiplication}$$

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Useful Rules - "The Properties of Real Numbers"

$$\left.\begin{array}{c}a+0=0+a=a\\3+0=0+3=3\end{array}\right\}\text{The addition property of zero}$$

$$\left.\begin{array}{c}a*0=0*a=0\\3*0=0*3=0\end{array}\right\}\text{The multiplication property of zero}$$

$$\left.\begin{array}{c}a*1=1*a=a\\5*1=1*5=5\end{array}\right\}\text{The multiplication property of one}$$

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Useful Rules - "The Properties of Real Numbers"

$$\left.\begin{array}{c}a+(-a)=(-a)+a=0\\4+(-4)=(-4)+4=0\end{array}\right\}\text{The inverse property of addition}$$

4 and -4 are called additive inverses

$$\left.\begin{array}{c}a*\frac{1}{a}=\frac{1}{a}*a=1, a\neq 0\\4*\frac{1}{4}=\frac{1}{4}*4=1\end{array}\right\}\text{The inverse property of multiplication}$$

4 and $\frac{1}{4}$ are called multiplicative inverses

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Useful Rules - "The Properties of Real Numbers"

$$\left.\begin{array}{c}a(b+c)=ab+ac\\3(4+5)=3*4+3*5\\3*9=12+15\\27=27\end{array}\right\}\text{The distributive property}$$

$$\left.\begin{array}{c}(b+c)a=ba+ca\\(4+5)2=4*2+5*2\\9*2=8+10\\18=18\end{array}\right\}\text{The distributive property}$$

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Division Properties of Zero and One

- Zero divided by any number other than zero is zero.

$$\text{For } a \neq 0,\ \frac{0}{a} = 0$$

$$\frac{0}{4} = 0$$

- Division by zero is not defined.

$$\frac{a}{0} \text{ is undefined}$$

$$\frac{4}{0} \text{ is undefined}$$

- Any number other than zero divided by itself is 1.

$$\text{For } a \neq 0,\ \frac{a}{a} = 1$$

$$\frac{-7}{-7} = 1$$

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Doing Math: Squares, Cubes, etc.

$$\begin{aligned} a^2 &= a \times a && \text{“a squared”} \\ a^3 &= a \times a \times a && \text{“a cubed”} \end{aligned}$$

etc.

The little number is called an *exponent*, or a “power”.

$$\begin{aligned} 2^2 &= 2 \times 2 = 4 \\ 10^2 &= 10 \times 10 = 100 \\ 10^3 &= 10 \times 10 \times 10 = 100 \times 10 = 1000 \end{aligned}$$

For 10’s, exponent represents number of 0’s to the right of 1

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Doing Math: Negative Powers

$$\begin{aligned} 2^{-2} &= \frac{1}{2^2} = \frac{1}{4} \\ 10^{-2} &= \frac{1}{10^2} = \frac{1}{100} = 0.01 \end{aligned}$$

for 10’s, exponent represents number of places to the right of 1, so if exponent is negative, decimal is to the left of 1.

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Doing Math: Square Roots

$$\begin{aligned} \sqrt{a} &= \text{“What number, when multiplied by itself, will result in } a\text{?”} \\ \sqrt{4} &= 2, \text{ because } 2 \times 2 = 4 \\ a^{\frac{1}{2}} &= \sqrt{a} \end{aligned}$$

How to compute square roots? On your calculator!

Practice (check your answers by squaring them):

$$\begin{aligned} \sqrt{5} &= \\ \sqrt{0} &= \\ \sqrt{0.045} &= \end{aligned}$$

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Doing Math: Scientific Notation

Scientific notation is a way to show (and do math with) very small or very large numbers

$$b \times 10^c$$

b = a decimal number between 1.0 and 9.99...

c = an exponent of 10

Example:

$$1.53 \times 10^3 = 1530$$

$$2.4 \times 10^{-2} = 0.024$$

To convert from scientific notation to standard notation, move the decimal c places to the right. (This is c places to the right if $c > 0$ and $|c|$ places to the left if $c < 0$).

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Doing Math: Scientific Notation

$$b \times 10^c$$

Multiplying 2 numbers in scientific notation:

- Multiply the b's
- Add the c's
- Round to the same or fewer decimal places than the least precise factor

Example:

$$(1.76 \times 10^{-8}) \times (3.2 \times 10^{12}) = (1.76 \times 3.2) \times 10^{(12-8)}$$

$$= 5.632 \times 10^4$$

3.2 is least precise factor, so round to 5.6×10^4

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Doing Math: Scientific Notation

$$b \times 10^c$$

Dividing 2 Numbers in Scientific Notation:

- Divide the b's
- Subtract the c's
- Round to the same or fewer decimal places than the least precise factor

Example:

$$(3.000 \times 10^9)/(2.1065 \times 10^{13}) = \frac{3.000}{2.1065} \times 10^{(9-13)}$$

$$= 1.4241... \times 10^{-4}$$

3.000 is least precise factor, so round to 1.424×10^{-4}

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Doing Math: Scientific Notation

$$b \times 10^c$$

Adding and Subtracting in Scientific Notation:

If the c's are the same:

- Add or subtract the b's
- Keep the same c
- Round to the same or fewer decimal places than the least precise factor
- Rescale if b is less than 1.0 or greater than or equal to 10.0

What if the c's are not the same?

- Realign to get the same c's, then add or subtract
- Use a calculator

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Doing Math: Scientific Notation

$$b \times 10^c$$

Examples:

$$1.53 \times 10^3 + 2.42 \times 10^3 = (1.53 + 2.42) \times 10^3 = 3.95 \times 10^3$$

$$2.4 \times 10^2 + 2.82 \times 10^2 = 5.22 \times 10^2$$

Round to number of digits in least precise number $\rightarrow = 5.2 \times 10^2$

$$4.78 \times 10^5 + 8.4 \times 10^5 = 13.2 \times 10^5$$

Keep b between 1.0 and $9.99\ldots$, so $\rightarrow = 1.32 \times 10^6$

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Algebraic Expressions: Order of Operations

Order is important!

Body Mass Index: $BMI = \frac{\text{Wt}}{\text{Ht}^2}$

For example, $BMI = \frac{67.5}{1.7^2} = 23.4$

We squared Ht *before* dividing Wt by the denominator.

What if we divided and *then* squared?

$$BMI \neq \left(\frac{\text{Wt}}{\text{Ht}}\right)^2 = \left(\frac{67.5}{1.7}\right)^2 = 1576.3$$

Not the same!

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Algebraic Expressions: Order of Operations

Solution: "Paul and Judi's Rules of Order"

1. Do operations in **P**arentheses first, starting with the innermost brackets.
2. Simplify **E**xponential expressions.
3. **M**ultiply and **D**ivide from left to right.
4. **A**dd and **S**ubtract from left to right.

"**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally"

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Algebraic Expressions: Order of Operations

Example:

$$\left[5 + \frac{(3+1)^2}{2} \times 2 - \frac{2}{1} + 1\right]^2$$

1. Innermost bracket $\left[5 + \frac{4^2}{2} \times 2 - \frac{2}{1} + 1\right]^2$
2. Exponents inside bracket $\left[5 + \frac{16}{2} \times 2 - \frac{2}{1} + 1\right]^2$
3. Multiplications and divisions inside bracket $[5 + 8 \times 2 - 2 + 1]^2$

$$[5 + 16 - 2 + 1]^2$$

4. Additions and subtractions inside bracket, left to right $[20]^2$
5. Exponents outside bracket $= 400$

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Practice: Order of Operations

$$\frac{\left[22 + (7-3)^2 + \frac{20}{4} \times 3 + \frac{27-2+3}{2}\right]^2}{2}$$

$$\frac{\left[22 + (4)^2 + \frac{20}{4} \times 3 + \frac{28}{2}\right]^2}{2}$$

$$\frac{\left[22 + 16 + \frac{20}{4} \times 3 + \frac{28}{2}\right]^2}{2}$$

$$\frac{\left[22 + 16 + 5 \times 3 + 14\right]^2}{2}$$

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Practice: Order of Operations

$$\frac{\left[22 + 16 + 15 + 14\right]^2}{2}$$

$$\frac{[67]^2}{2}$$

$$\frac{4489}{2}$$

$$2244.5$$

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Functions

A **function** is a rule or procedure such that when you give it a number, it gives you back one and only one number. For example, the Ideal Body Weight function:

You give it height → [IBW function] → It gives you IBW

↑ This is the "argument" of the function: what you put in

↓ (IBW) = 106 + 6($h-60$)

↑ This is the "value" of the function: what you get out

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Functions

Another function:

You give it gestational age, toxemia → [Predicting Length of newborn function] → It gives you length

↑ arguments

↓ length = $6.61 + 1.05g - 3.48t + 0.06gt$

↑ value

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Graphs of Functions

We can call the “value” of a function y and the “argument” of a function x. For every x, we can determine y, and we can plot the points.

$$y = 2x + 1$$

x	y
-2	-3
-1	-1
0	1
1	3
2	5

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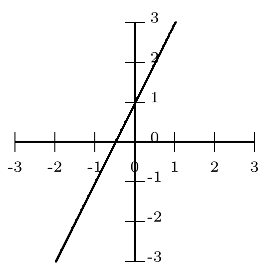
Graphs of Functions

$$y = 2x + 1$$

x	y
-2	-3
-1	-1
0	1
1	3
2	5



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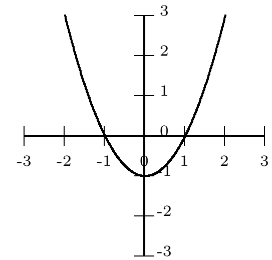
Graphs of Functions

$$y = x^2 - 1$$

x	y
-2	3
-1	0
0	-1
1	0
2	3



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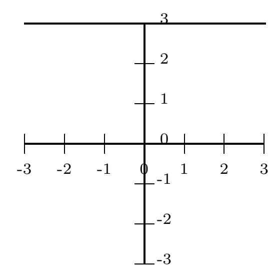
Graphs of Functions

$$y = 3$$

x	y
-2	3
-1	3
0	3
1	3
2	3



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Linear Functions

A **linear** function is a function whose graph is a straight line. It has the following form (called the slope-intercept form):

$$y = mx + b$$

where m is the **slope** and b is the **y-intercept**, or constant term.

What does the slope mean?

- How steep the line is
- How much y changes with every unit change in x

What does the y-intercept mean?

- Where the line crosses the y-axis
- The value of y if $x = 0$

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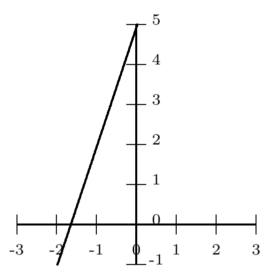
Linear Functions

Examples of linear functions:

$$y = 3x + 5$$

x	y
-2	-1
-1	2
0	5
1	8
2	11



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Linear Functions

Examples of linear functions: Ideal Body Weight for Males

We use y to represent the *value* of the function and x to represent the *argument* of the function. Here, y = ideal body weight and x = height in inches.

$$y = 106 + 6(x - 60)$$

$$y = 106 + 6x - 360$$

$$y = 6x - 254$$

slope = 6: For every one-inch increase in height, there is a 6 lb increase in ideal body weight.

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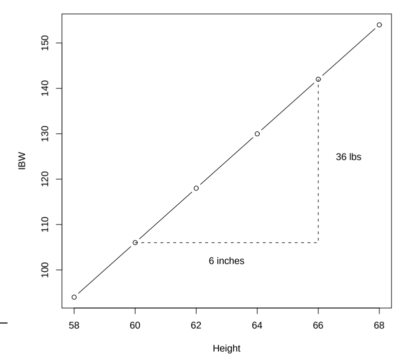
Linear Functions



x	y
0	-254
60	106
66	142

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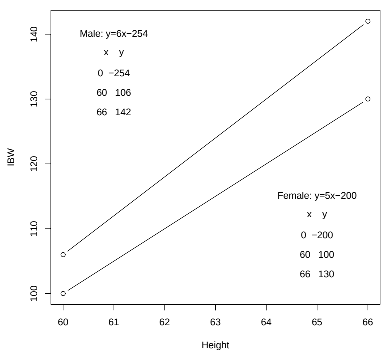
Linear Functions

Compare IBW graphs for males and females:



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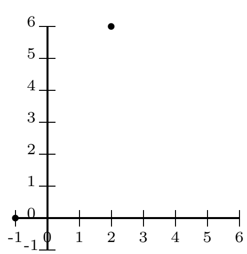
Example: Find the Equation for a Line

Suppose you have two points (-1,0) and (2,6) and you want to find the equation for the line through those two points.



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Example: Find the Equation for a Line

1. The slope is the change in y divided by the change in x:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{2 - (-1)} = \frac{6}{3} = 2$$

2. Set up a formula in the standard form, using the slope you calculated

$$y = 2x + b$$

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Example: Find the Equation for a Line

3. Substitute the x and y values for one of the points (2,6) and solve for b

$$6 = 2(2) + b$$

$$6 = 4 + b$$

$$6 - 4 = b \Rightarrow b = 2$$

$$y = 2x + 2$$

4. Check your answer using the x and y values for the other point

$$0 = 2(-1) + 2? \text{ Yes!}$$

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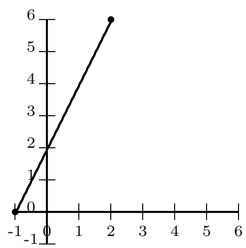
Example: Find the Equation for a Line

$$y = 2x + 2$$



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Another Way to Find the Equation for a Line

We have been using the slope-intercept form of the equation for a line. Another way to find the equation for a line is to use the point-slope method.

$$y - y_1 = m(x - x_1)$$

m = slope

(x_1, y_1) = one point on the line

So, given slope = 2 and point $(x_1, y_1) = (-1, 0)$:

$$y - 0 = 2(x - (-1))$$

$$y = 2x + 2$$

Same as before!

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Application: Finding Slopes

You have a new drug for prostate cancer that works (you think) by stopping the cancer's growth, but not by killing existing cancer cells. You are following two patients whose only sign of cancer is a rising level of prostate-specific antigen (PSA). You measure the patients' PSA repeatedly, and as soon as it rises above 10 ng/dl, you start them on the experimental drug. Here is what happened to two of your patients. Which one had the best response to the drug?

Patient 1			Patient 2	
Date	PSA (ng/dl)		Date	PSA (ng/dl)
Jan 1	3		Feb 1	6
Aug 1	13	drug started	June 1	20
Dec 1	12		Sept 1	15

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Application: Finding Slopes

Recall that we defined slope as how much y changes with a unit change in x. This can be thought of as

$$\frac{\Delta y(\text{change in } y)}{\Delta x(\text{change in } x)}$$

Let's convert the dates in our table to month numbers, and let's consider x to be time and y to be PSA level. We can then compute the slope of the PSA from baseline to treatment, and from treatment to first follow-up.

Patient 1				Patient 2		
Date	Time	PSA		Date	Time	PSA
Jan 1	1	3		Feb 1	2	6
Aug 1	8	13	drug started	June 1	6	20
Dec 1	12	12		Sept 1	9	15

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Application: Finding Slopes

Patient 1, Change from baseline to start of treatment

$$\frac{\Delta y}{\Delta x} = \frac{13-3}{8-1} = \frac{10}{7} = 1.43$$

Patient 1, Change from start of treatment to follow-up

$$\frac{\Delta y}{\Delta x} = \frac{12-13}{12-8} = \frac{-1}{4} = -0.25$$

Patient 2, Change from baseline to start of treatment

$$\frac{\Delta y}{\Delta x} = \frac{20-6}{6-2} = \frac{14}{4} = 3.5$$

Patient 2, Change from start of treatment to follow-up

$$\frac{\Delta y}{\Delta x} = \frac{15-20}{9-6} = \frac{-5}{3} = -1.67$$

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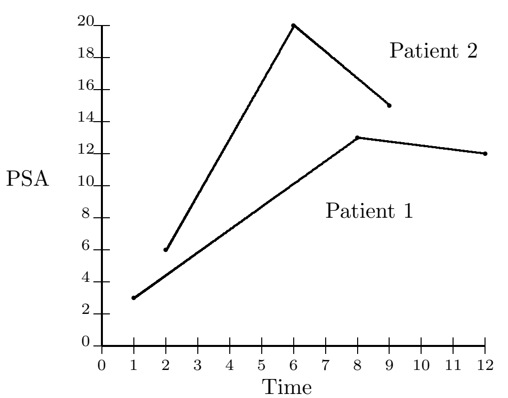
Application: Finding Slopes



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Solving Quadratic Equations

You may need to find the solution to a quadratic equation. To do this, you must use the distributive, additive, and multiplicative properties to get the equation into this form:

$$ax^2 + bx + c = 0$$

Then you can plug a, b, and c into the following equation, which is called the **quadratic formula**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\sqrt{b^2 - 4ac}$ is called the *discriminant*.

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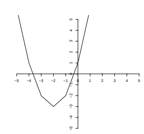
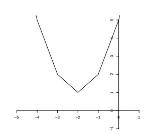
Solving Quadratic Equations

The solution to a quadratic equation specifies where it crosses the x axis. A quadratic equation may have 2 solutions:



A quadratic equation may have no solutions:



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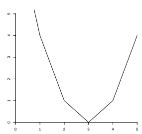
Solving Quadratic Equations

A quadratic equation may have one solution:



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Solving Quadratic Equations: Example

$$
\begin{aligned}
3(x^2+2x)+2 &= -x^2-6x-4 \\
3x^2+6x+2 &= -x^2-6x-4 \\
4x^2+6x+2 &= -6x-4 \\
4x^2+6x+6 &= -6x \\
4x^2+12x+6 &= 0
\end{aligned}
$$

So $a = 4$, $b = 12$ and $c = 6$.

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Solving Quadratic Equations: Example

$$a = 4, b = 12, c = 6$$

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
x &= \frac{-12 \pm \sqrt{12^2-4(4)(6)}}{(2)(4)} \\
x &= \frac{-12 \pm \sqrt{48}}{8} \\
x &= \frac{-12}{8} \pm \frac{\sqrt{48}}{8} \\
x &= -1.5 + 0.8660 = -0.634 \\
x &= -1.5 - 0.8660 = -2.366
\end{aligned}
$$

The two solutions are -0.634 and -2.366.

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Solving Quadratic Equations: Example

Check the solutions:

$$
\begin{aligned}
4x^2+12x+6 &= 0 \\
4(-0.634)^2+12(-0.634)+6 &= 1.608-7.608+6=0 \\
4(-2.366)^2+12(-2.366)+6 &= 22.392-28.392+6=0
\end{aligned}
$$

Good!

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Solving Quadratic Equations: Example

We can graph quadratic equations in a manner similar to that for linear functions:

$$y = 4x^2 + 12x + 6$$

x	y
0	6
-1	-2
-2	-2
-3	6
-0.634	0
-2.366	0

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Graphing Quadratic Equations